Speaker: Chris Kapulkin, University of Western Ontario

Mini-Course Title: Discrete homotopy theory

Time: Friday, Nov 10th, 9:30am
Monday, Nov 13th, 9:30am
Wednesday, Nov 15th, 9:30am

Abstract: Discrete homotopy theory, introduced around 20 years ago by H. Barcelo and collaborators building on the work of R. Atkin from the mid seventies, is a homotopy theory of (simple) graphs. As such, it applies techniques previously employed in the "continuous" context to study discrete objects. It has found applications both within and outside mathematics, including: matroid theory, hyperplane arrangements, topological data analysis, and time series analysis.

The first part of this mini-course is an introduction to discrete homotopy theory. We define the main concepts, including homotopy of graph maps and homotopy groups of a pointed graph. We also briefly discuss some of the applications of the field.

The second part discusses the recently introduced "cubical setting" for discrete homotopy theory, which brings a whole host of new tools into the field, including the topological realization theorem (formerly a conjecture of E. Babson, H. Barcelo, R. Laubenbacher, and M. de Longueville from 2006), the long exact sequence of a fibration, and the Hurewicz theorem. It is based on joint work with D. Carranza (arXiv:2202.03516).

The third part zooms in on the notion of the fundamental group and the fundamental groupoid of a graph. Several computational tools are discussed, including the Seifert-van Kampen theorem and the theory of covering graphs. It is based on joint work with U. Mavinkurve (arXiv:2303.06029).

Speaker: Teresa Hoekstra Mendoza, CIMAT

Title: Configuration spaces of graphs

Time: Monday, Nov 13th, 11:00am

Abstract: Given any topological space, in particular a graph G, we can define its (ordered) configuration space on n points as follows. Consider the product GxGx...xG n times and remove every tuple in which two or more entries are the same. In this talk I shall give a few examples of configuration spaces of graphs on two points, and a description of the homotopy type of the 1-skeleton of configuration spaces of trees on n points. I will also mention no k-equal spaces, which are like configuration spaces but instead of removing all tuples in which two or more entries are the same, we only remove tuples in which k or more entries are the same for k<n.

Speaker: Julian Candela, CIMAT

Title: Central Limit Theorems for Large Random Simplicial Complexes

Time: Monday, Nov 13th, 12pm

Abstract: Stochastic topology is the study of the topological properties of randomly formed spaces. In this area, we can find several articles dedicated to the study of homology for different models of random simplicial complexes. One of the most important models was introduced by Costa and Faber in their article "Large random simplicial complexes" where they described and studied a model of random simplicial complexes that are random in every possible dimension. For this presentation, we use a normal approximation theorem by Stein's method to prove central limit theorems for the random simplicial complexes model by Costa and Farber.

Speaker: Daniel Carranza, Johns Hopkins University

Title: Hurewicz Theorem in Discrete Homotopy Theory

Time: Tuesday, Nov 14th, 10:00am

Abstract: Homotopy groups, while easy to define, are notoriously hard to compute (both for spaces and graphs). By contrast, homology groups are much more computable, despite their more complicated definition. One would like a way of leveraging homological techniques to compute homotopy groups, and in classical algebraic topology, this is achieved by means of the Hurewicz theorem. This theorem states that for a connected space, the first non-trivial homotopy and homology groups agree (up to abelianization). I will present a discrete analogue of this theorem, and show how it can be used to compute previously-unknown higher homotopy groups of graphs. This talk is based on joint work with Chris Kapulkin (arXiv:2202.03516).

Speaker: Maru Sarazola, University of Minnesota

Title: A homotopical framework for path homology of directed graphs

Time: Tuesday, Nov 14th, 11:00am

Abstract: Traditionally, the way we compare mathematical structures is by using the notion of equality, or even of isomorphism. However, there are many settings where this is no longer the natural notion of "sameness". A notable example occurs when our objects admit a homology construction: then, we want two objects to be "the same" if they have identical homology.

Homotopy theory provides the framework required to work in these settings. In this talk, we will describe an algebraic invariant of directed graphs called "path homology", and introduce a new homotopical framework that encodes "sameness up to path homology". We will also show how this new framework allows us to make homology-invariant constructions.

Speaker: Laura Scull, Fort Lewis College

Title: X-homotopy for graphs and Z/2-equivariance

Time: Tuesday, Nov 15th, 12:pm

Abstract: X-homotopy for graphs is defined using the categorical product of graphs. In this context, our graphs are rigid and edges cannot be collapsed. Thus there is a sense in which graphs in this theory are built out of edges, instead of vertices. The 'flip' which reverses direction of an edge introduces a Z/2-action in this wordview, and X-homotopy phenomena such as parity can be then understood as Z/2-equviariant features.

In this talk, I will focus on the X-fundamental group. I will introduce this fundamental group in a classical way, as homotopy classes of paths in our graph, and give examples (and pictures). Then I will show how taking an edge-centric approach allows us to define a fundamental group as a Z/2-equivariant fundamental group of paths of edges. I will motivate the equivariant theory and then explain this second definition. These two seemingly different definitions give equivalent groups, providing a way of seeing the parity showing up in the traditional definition as coming from a Z/2-action.

Speaker: Gregory Lupton, Cleveland State University

Title: A Second Homotopy Group in Digital Topology

Time: Wednesday, Nov 15th, 11:00am

Abstract: (Joint work with Musin, Scoville, Staecker and Trevino)

Digital topology refers to the use of notions and methods from (algebraic) topology to study digital images. A digital image in our sense is an idealization of an actual binary digital image, which consists of pixels in the plane, or higher dimensional analogues of such. One aim of digital topology is to provide useful theoretical background for certain steps of image processing, such as feature extraction or recognition.

An extensive literature on digital topology includes several treatments of the fundamental group. I will report on our recent work in which we develop a notion of a second (higher) homotopy group and calculate its value to be **Z** for a digital image that may reasonably be interpreted as a digital 2-sphere. This calculation involves some interesting combinatorial ingredients. Our development and calculation may equally well be applied in the (graph-theoretic) settings of tolerance spaces and X-homotopy theory. The definitions readily extend to higher homotopy groups of any dimension.

I will also speculate briefly about possible applications of our work.